**Chapter 1: Complex Numbers**

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There are certain equations, such as , which have no real solutions. To deal with these, we need to start considering the set of complex numbers.

A complex number has the form , where and and are both real numbers. is called the **imaginary unit**. is called the **real part** of , denoted as , and is called the **imaginary part** of , denoted as . itself is called a **complex variable**.

Real numbers are actually a subset of complex numbers, where . If, on the other hand, but , then the complex number is called a **pure imaginary number**.

The **complex conjugate**, or simply the **conjugate**, of a complex number is . This is denoted by or .

## 1.4 Fundamental Operations

Two complex numbers, and , are considered equal only if **and** .

For **addition** and **subtraction**, we simply add and subtract the real parts with each other and the complex parts with each other.

For **multiplication**, we perform ordinary multiplication.

**Division** becomes a little more difficult. We need to make the denominator a real number, and to do this, we need to multiply both the denominator and the numerator by the conjugate form of the denominator.

## 1.5 Absolute Values

The **modulus** or **absolute value** of a complex number, , is defined as

This can also be denoted as or .

When dealing with absolute values, there are some properties which can make things easier for us:

1. , given that

## 1.7 Graphical Representation

A complex number can be represented with a 2D graph, with the real part on the -axis and the complex part on the -axis. Thus, the complex number would be units on the -axis and -units on the -axis.

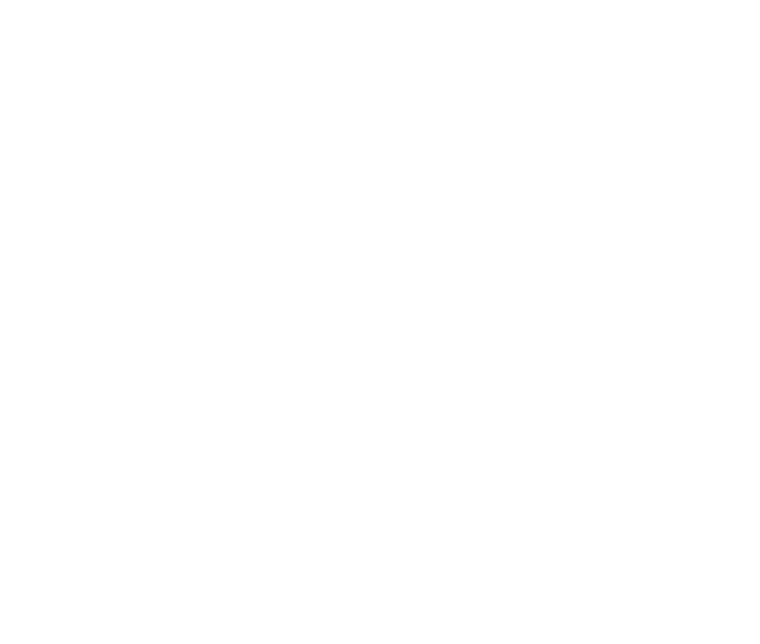
If we consider this point to be , then , or , the distance from the origin, is just the absolute value of .

In polar form, can be written as , where is called the **amplitude** or **argument** and is denoted as .

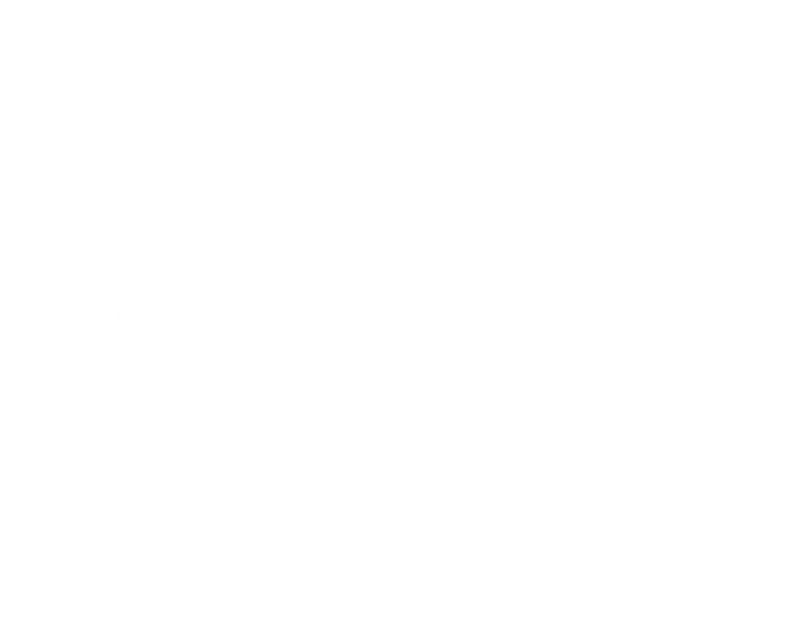
For two complex numbers, and , the distance between and is given by

The plane on which we represent complex numbers is called the **complex plane** or the **Argand Diagram**.

We can also show arithmetic operations on complex numbers graphically.



For subtraction, we would add the first number with the complex conjugate form of the second number.



## 1.9 De Moivre’s Theorem

Let and .

Generalizing this,

If , this becomes

## 1.10 Roots of a Complex Number

A number is called the th root of a complex number if . We can write .

De Moivre’s Theorem tells us that

Based on this,

We also know that

and

for different values of .

Thus,

This tells us that there are different roots for , provided that . The roots can be found using the different values of .

## 1.11 Euler’s Formula

To be able to understand Euler’s Formula, we first need to know three series:

Using these,

For the complex number ,

In the special case where , this reduces to .

In terms of De Moivre’s Theorem,

## 1.13 The th Roots of Unity

The solutions for the equation , where is a positive integer, are called the th roots of unity. They are given by